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Field-controlled spectra and entanglements in two-electron double-barrier nanorings

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Abstract

A two-dimensional double-barrier nanoring under in-plane electric fields is proposed to investigate field-controlled two-electron spectra and entanglements. It has been found that changing the direction of the field can destroy the symmetries of the wavefunctions and has great impact on the spectra. The angle between the barriers, along with the intensity and direction of the field, can be used for controlling the entanglements of both the ground and excited states. The structure-dependent and field-controlled far-infrared spectroscopies are useful in such investigations. It is helpful for manipulating the entanglements of indistinguishable particles in nanostructures.

(Some figures in this article are in colour only in the electronic version)

In current theoretical and experimental investigations of solid-state quantum computation, an important subject is to utilize the tunable structural parameters of nanostructures and external fields to implement quantum manipulations. The indistinguishability of particles within the nanoscale may have great effects in multi-qubit manipulations [1, 2] and the quantum encoding scheme [3]. It is also a foundation for understanding the entanglement behaviours of charged exciton states [5]. But relevant investigations are still very limited. To study the characters and control of entanglement of identical particles in nanostructures [6], a two-dimensional double-barrier nanoring is proposed and investigated in this paper.

There has been great attention paid to the electronic and optic properties of the ring-like quantum dot, the so-called nanoring [7, 8]. The Aharonov–Bohm effect within the ringlike structure has been widely studied from both theoretical and experimental perspectives [7, 9]. Besides these, it has been shown that additional structures, such as two barriers, can bring new characters to the system [10]. To a certain extent, such a system can be viewed as coupled quantum dots (CQDs) [11, 12], with a multiply connected domain. However, compared with ordinary vertical or lateral CQDs, the shape, size and field-assisted control of the electronic

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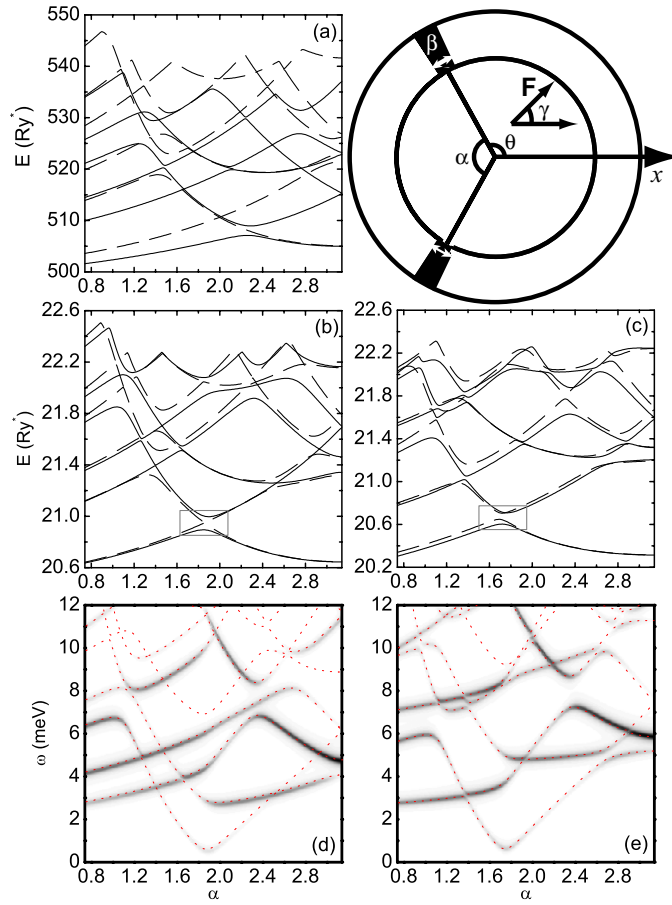


Figure 1. The α -dependence of two-electron singlet (solid lines) and triplet (dashed lines) energy levels in a double-barrier nanoring for $R = 0.4a_B^*$, $W = 0.2a_B^*$ and $V_0 = 500 \text{ Ryd}^*$ with $F = 0$ (a) and $R = 2a_B^*$, $W = a_B^*$ and $V_0 = 20 \text{ Ryd}^*$ with $F = 0$ (b) and with $F = 0.4F_0$ of $\gamma = \pi/2$ (c), respectively. The ω -dependent FIR absorption coefficient (d) and (e) as a function of α (arbitrary units with linear scales) corresponding to the parameters of (b) and (c). The dotted lines outline the traces of the resonance energies. The darker the grey, the larger the absorption coefficient.

states and entanglements in a double-barrier nanoring may be more facilitated due to the narrow barriers in the ring. We will explain how to modify the entanglements of two identical electrons in such a system by choosing appropriate structural parameters and adjusting the electric fields. Furthermore, spectral analysis is an important experimental technique in studies of nanostructures. Like photoluminescence spectra, which are widely adopted in exciton [4] and charged exciton systems [5], far-infrared spectroscopy, which has great application in the studies of electronic properties in CQDs [13, 14] and nanorings [7], is used in the analysis of two-electron entangled states and their control.

A two-dimensional nanoring with two identically sectorial barriers is schematically shown in figure 1. The Hamiltonian of two electrons in a system under an electric field (\mathbf{F}) in effective atomic units can be written as

$$H = \sum_{i=1,2} (-\nabla_i^2 + V_{ci} + V_{gi} + \mathbf{F} \cdot \mathbf{r}_i) + \frac{2}{|\mathbf{r}_1 - \mathbf{r}_2|}. \quad (1)$$

V_c is the hard wall potential, which is 0 in the ring and infinite elsewhere. The V_{gi} are the barriers in the ring whose heights are V_0 in the barriers and 0 elsewhere. The width of the barrier is selected to be quite small to ensure that the wavefunction has maximally one angular node inside each barrier. α , β and γ are the angle between two barriers, the flare angle of each barrier and the angle between the direction of \mathbf{F} and the x -axis, respectively. The energy and length units are the effective Rydberg $\text{Ryd}^* = m_e^* e^4 / 2\hbar^2 (4\pi\epsilon_0\epsilon_r)^2$ and the effective Bohr radius $a_B^* = 4\pi\epsilon_0\epsilon_r\hbar^2 / m_e^* e^2$. The unit of \mathbf{F} is $F_0 = \text{Ryd}^* / ea_B^*$. For InAs/GaAs materials, for example, $\text{Ryd}^* = 5.8 \text{ meV}$, $a_B^* = 10 \text{ nm}$ and $F_0 = 5.8 \text{ kV cm}^{-1}$.

The eigenvalues and eigenfunctions of H can be solved by the exact diagonalization method with the use of the eigenfunctions of an ideal nanoring without barriers as a set of single-particle basis. Then the FIR absorption coefficient of circularly polarized light can be calculated within the electronic dipole approximation [15].

The barriers separate the ring into spatial left and right segments: L for $\theta \in [\pi - (\alpha/2), \pi + (\alpha/2)]$ and R for $\theta \in [-\pi + (\alpha/2), \pi - (\alpha/2)]$, respectively. Each eigenstate Ψ of H can be projected to the form

$$|\Psi\rangle = \sum_{\mu, \nu=0}^3 P_{\mu\nu} c_\mu^\dagger c_\nu^\dagger |0\rangle \quad (2)$$

where c_μ^\dagger (c_ν^\dagger) and c_μ (c_ν), $\mu, \nu = L \uparrow, L \downarrow, R \uparrow, R \downarrow$, are fermionic creation and annihilation operators. \uparrow and \downarrow stand for one-particle states with $S_z = 1/2$ and $-1/2$, respectively. Then the von Neumann entropy can be used for quantifying the entanglements between two identical electrons and obtained by

$$S = -\text{Tr } \rho \log_2 \rho \quad (3)$$

where $\rho = 2PP^\dagger$ is the reduced density matrix of a randomly chosen particle. It can be also calculated from the explicit formula [16]:

$$S = 1 - x \log_2 x - (1 - x) \log_2 (1 - x) \quad (4)$$

where

$$x = \frac{1}{2} \left(1 + \sqrt{1 - \eta^2} \right) \quad (5)$$

and

$$0 \leq \eta \equiv 8|P_{01}P_{23} - P_{02}P_{13} + P_{03}P_{12}| \leq 1. \quad (6)$$

For two identical particles, $S = 1$ and 2 correspond to the unentangled and maximum entangled states [16–18], respectively. In our case, the spatial parts of the two-electron eigenstates can be separated from the spin ones. It can be demonstrated that if the leading component of the spatial part is an equal-weighted superposition of inter-segment components ($|LR\rangle$ and $|RL\rangle$) or intra-segment ones ($|LL\rangle$ and $|RR\rangle$), and the spin part is a singlet or triplet with $S_z = 0$, the entanglement will reach a maximum. The other two triplets with $S_z = \pm 1$ are always unentangled, and we will not discuss them in this paper.

The α -dependent energy spectra of the nanorings with different sizes are shown in the left-hand column of figure 1. It has been demonstrated that the single-particle spectra have similar structure if the area of the ring is scaled to n times and V_0 and F are scaled to V_0/n and $F/n^{3/2}$, respectively [10]. However, with the same scale, the two-particle spectra strongly depend on the size of the ring. This is because the Coulomb energy is more dominant than the single-particle energy in a larger ring, but just the reverse is true in a smaller one. In the following discussion on spectra and entanglements, we will take $R = 2a_B^*$, $W = a_B^*$ and $V_0 = 20 \text{ Ryd}^*$, which are reasonable values to be used.

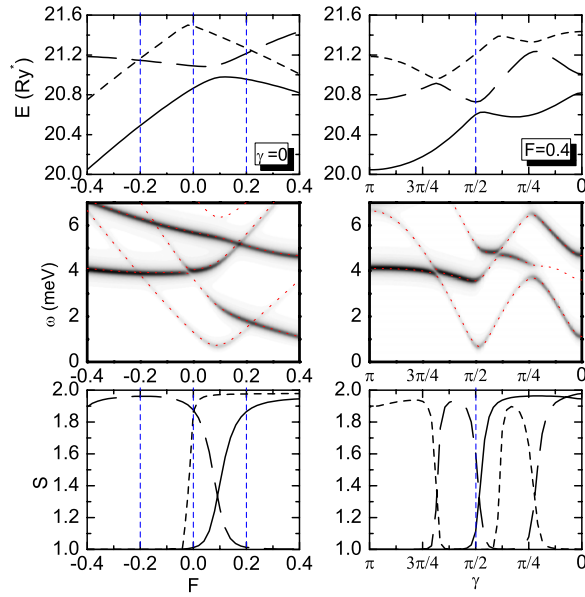


Figure 2. Energy levels (upper row) and entropies (middle row) of the first three singlets, and ω -dependent FIR absorption coefficient (lower row) of the first several states with $\alpha = 1.69$ as functions of F with $\gamma = 0$ (left-hand column) and γ with $F = 0.4F_0$ (right-hand column), respectively.

There are crossings and anticrossings between different energy levels in the spectra. These two different behaviours originate from the symmetries of the states. If the electric fields are absent or applied along the x -axis, the Hamiltonian is mirror symmetric about the x -axis no matter what the value of α is. Then the eigenstates should be either even or odd symmetric about the same axis. The anticrossings imply the admixture of two states when changing α , so they can only occur between states with the same symmetry. States with different symmetries can only cross each other in order to preserve their respective symmetries. These crossings and anticrossings may be identified by the FIR spectroscopies. The ω -dependent FIR absorption coefficient as a function of α with the same condition in figure 1(b) is presented in figure 1(d). As the ground state is always a singlet for any α , there are only resonance energies corresponding to singlets. If there is a crossing (anticrossing) between two singlets in the spectra, the corresponding FIR resonance energies will also have a crossing (anticrossing) with the same α . An interesting feature of the spectroscopies is that the absorption coefficients corresponding to certain states change gradually near a crossing, but have sharp variation and exchange their intensities near an anticrossing (except the anticrossing between the ground state and the second singlet). This is just the evidence of the admixture of two states in the vicinity of an anticrossing.

Furthermore, if $\gamma \neq 0$ or π , the eigenstates need not have certain mirror symmetries and there will be no crossings in the spectra and FIR spectroscopy results which have been shown in figures 1(c) and (e). Such an alternation will have great impact on the control of the entanglements.

The anticrossings imply the admixture of the states and they accompany the remarkable change of entanglement entropies. So it makes the control of entanglements easier to choose the appropriate α where there is an anticrossing between the required states. Based on this principle, we have chosen $\alpha = 1.69$ in figure 2 to investigate the methods for controlling

the entanglements of the first three singlets. The entropies of the states which are used for quantifying the entanglements are calculated by equation (4). As is shown in the left-hand column of figure 2, an electric field along the x -axis with quite a low intensity is enough to modify the entanglements because there are just anticrossings between different states. As F changes from 0 to 0.2, the admixture of the ground state and the second singlet makes their entropies increase and decrease respectively. This is a process of admixture of the states with leading components $|RR\rangle$ and $|LR\rangle + |RL\rangle$. There is also an anticrossing between the third and fourth singlets (not shown here) and their admixture also results in the variation of the entropies.

Not only can adjusting the intensity modify the entanglements, but also changing the direction of the field may achieve the same goal. In the right-hand column of figure 2, we show the changes of the entropies during half a variation period of γ with $F = 0.4F_0$. Compared with adjusting the intensity of the field with $\gamma = 0$, changing γ with fixed F has the similar effect on the entropy of the ground state. However, there are two extra anticrossings instead of the two crossings between the second and third singlets. Combined with the other two anticrossings in the spectra, the variations of their entropies are no longer monotonic in half a period. In fact, the situation for the first two triplets is more apparent. In figure 1(b), it can be imagined that their entropies are hardly modified if α is also chosen to be 1.69 since there is only a crossing and they cannot mix with each other. However, as shown in figure 1(c), the crossing has become an anticrossing when $\gamma \neq 0$. This alternation will undoubtedly provide the possibility to modify the entanglements more easily.

In all the cases discussed above, low intensity fields can control the entanglements sensitively and avoid the breakdown of the device. All these features come from the proper choice of α . It can be imagined that F has to be much larger if α is chosen to be close to π . The F -dependent spectra with crossings and anticrossings can be also observed in the FIR absorption spectroscopy results which have been presented in figure 2. Then the corresponding relation between the FIR spectroscopy and the energy spectra can help to confirm the positions of anticrossings and the choice of proper structural parameters in future experimental investigations of solid quantum computation.

The three-dimensional diagram of the ground state's and the fifth state's entropies calculated from equation (4), as functions of the structural and external fields parameters, are plotted in figure 3. The values of α have been selected carefully according to the criteria discussed above. For the ground state, the value of α should be near 1.7, where the two segments of the ring are very unequal in size. This requirement is easy to meet in our scheme but may be more difficult in CQD systems. An improper α will make the control of the entanglement difficult or even impossible. For example, it can be seen in figure 3(c) that for the fixed $F = 0.2F_0$, adjusting γ cannot make the entropy higher than 1.2 if α is close to 1.5, which is too small for the ground state. If a higher entanglement is required, the intensity of the field has to be increased. It is also difficult to modify the entanglement of the ground state if α is chosen too close to π , where the unentangled state is hard to obtain.

In contrast to the ground state, the value of α for the fifth singlet must be very close to π to ensure that the inequality of the two segments is not very severe. It can be seen from figures 1(b) and (c) that there are indeed anticrossings between the fifth and fourth singlets near such α . Then adjusting the intensity or direction of the field can change the two electrons from an unentangled to an almost maximum entangled state and vice versa. It can be seen in figure 3 that the variation of the entropies of the ground state and the fifth singlet are different in shape. The entropy of the ground state can be kept on a high level for a large range of the parameters. In contrast, when the entanglement of the fifth singlet reaches the maximum, it will fall off rapidly. These different behaviours indicate that the electric fields must be tuned very accurately in the manipulation of the entanglement of the fifth singlet.

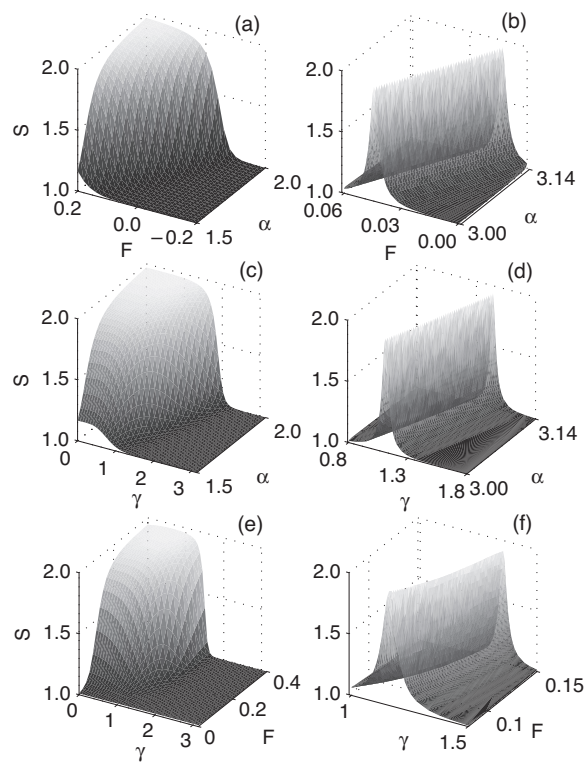


Figure 3. The entropies of the ground state (left-hand column) and the fifth singlet (right-hand column) as functions of α and F with $\gamma = 0$ ((a), (b)), α and γ with $F = 0.2F_0$ (c) and $F = 0.1F_0$ (d), and F and γ with $\alpha = 1.69$ (e) and $\alpha = 3.04$ (f), respectively.

In summary, we have proposed to employ a double-barrier nanoring to investigate two identical electrons' spectra and entanglements. It has been found that the α -dependent energy spectra are remarkably affected by the size and the symmetry of the ring. In order to control the entanglements more conveniently and sensitively, the value of α should be carefully selected to be near an anticrossing of the required states. Then moderate in-plane electric fields can be used for modifying the entanglements. In different ranges of α , the entanglements of both the ground state and some excited states can be well controlled by F or γ . FIR absorption spectroscopy can identify anticrossings and help to choose proper values for α , F and γ . These results reveal some universal properties of entanglements in nanostructures and should be useful in understanding the entanglements in the system of identical particles, such as electrons or charged excitons in CQDs.

Acknowledgments

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